Topic:Idea of de Broglie Waves
Course:B.Sc part 3 Chemistry (Hons.)
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In 1924, on the basis of Einstein's special relativistic relation between energy and mass namely $E = mc^2$ argued that micro particles of matter like electrons should exhibit wave-particle duality in a similar way to that of photon – the particle of electromagnetic radiation. By 1924, the wave-particle duality of electromagnetic radiation was well established through photo-electric effect and Compton's experiments. De-Broglie argued that the relation $E = mc^2$ suggests that a particle of mass" m" can be destroyed and converted into energy "E" of pure electromagnetic radiation. Now, if electromagnetic radiation can have wave-particle duality, then the particle of mass "m" which was converted into radiation should have the same wave particle duality. The reason is that mere transformation from particle to radiation should not develop new properties in latter, which was not there in the former. Hence, the well established fact that electromagnetic radiation exhibit wave-particle duality implies that particles like electrons should exhibit same type of wave-particle duality. This means that the energy and momentum of electron should be given by same equations as that of photon, namely $E = \boldsymbol{h\nu}$ and $\lambda = \boldsymbol{h/p}$ where $\boldsymbol{p}$ is the momentum of the particle and $\boldsymbol{\nu}$ and λ are the frequency and wavelength of the associated matter wave respectively.

Now, $\boldsymbol{\nu} = E/\boldsymbol{h} = mc^2/\boldsymbol{h}$ .....(1)

And $\lambda = \boldsymbol{h/p} = \boldsymbol{h}/mv$ .....(2)

λ is called the De –Broglie wavelength and equation (2) is called De-Broglie equation. The phase velocity of De-Broglie waves is given by

$\boldsymbol{v}_p = \boldsymbol{\nu}\lambda = mc^2/\boldsymbol{h} \times \boldsymbol{h}/mv = c^2/v$ .....(3)

Since the velocity v must be less than the velocity of light c, the de-Broglie waves travel faster than velocity of light.

(1) Except matter waves, all other waves observed in nature, are described by same differential equation called wave equation. This wave equation is real and second order in space co – ordinates and second order in time co – ordinate. Matter waves, on the other hand, obey Schrodinger equation which is complex, first order in time co – ordinate and second order in space co – ordinates.

(2) Except matter waves, all other waves can be described by real mathematical function. Matter waves, on the other hand, in general, can only be described by a complex function.

(3) All waves propagating freely in space (Vacuum) can be described by a plane wave characterised by an amplitude factor and phase factor. Phase factor of all waves, is same, namely, $(kx - \omega t)$ in one dimension. Matter waves involve complex function of the phase factor. On the other hand, "all other waves", involves only the real function of phase factor. The amplitude of "all other waves" is real, hence, measurable but the amplitude of matter waves is, in general, complex, and hence, cannot be measured.

(4) "All other waves" have a particular physical source which generates the wave. But matter waves have no such particular physical source. Matter waves get associated with respective particles due to their motion.

Note:- A particle, absolutely at rest, according to Heisenberg's uncertainty principle, cannot be located at a definite position. Its de-Broglie wavelength is infinite, phase velocity is also infinite and therefore, is physically meaningless.

(5) Physical meaning of these complex matter waves is obtained by interpreting $|\psi(\vec{r},t)|^2$ - the absolute square of the wave

function ψ ($\vec{r}$, t), which describes the state of motion of the particle. The Interpretation is that, $|\psi(\vec{r},t)|^2$ is the probability of finding the particle in a unit volume constructed around the position $\vec{r}$ at time t.

(6) That, the phase velocity of these matter waves exceeds the velocity of light c, does not contradict special theory of relativity as it is not a real physical signal.

Experimental verification of existence of matter waves (de – Broglie waves)

The experimental verification of existence of matter waves came from two different experiments of diffraction of electron beams. (1) One by Davission and Germer by diffracting electron beams of different kinetic energies such as 44 ev., 54 ev. and so on from Nickel crystal. The de-Broglie wave length of 54 ev. Electrons are found from de-Broglie formula to be 1.66 Angstrom units. The measured value came out to be 1.65 Angstrom units. Second experiment came from G.P. Thomson who diffracted electron beams gold foil of thickness 10^{-8} m. He obtained X-ray like diffraction pattern on a photographic plate. Measured wavelength matched with the theoretically predicted value.